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NRL Report 7379
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Further Results on Estimation of Ship Parameters

[Unclassified Title]

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January 26, 1972

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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CONTENTS

ABSTRACT
[Secret]

A method has been devised which is capable of estimating a ship's heading and length with a noncoherent side-looking radar possessing two beams, one squinted forward and the other aft. This method uses the ship's projections on the two squinted beams for the estimation. Unfortunately, besides the correct estimate, three spurious pairs of estimates are given. This ambiguity is removed by estimating the target's position in each squinted beam and then using the target's estimated velocity, which is derived from the two positions, to select one of the four estimates. Then, by using a Monte Carlo method, results are obtained on the accuracy of the estimation method. For a typical destroyer with a 450-ft length, the standard deviations of the errors in length and heading are approximately 30.0 ft and 14.0°.

AUTHORIZATION

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Manuscript submitted December 7, 1971.

FURTHER RESULTS ON ESTIMATION OF SHIP PARAMETERS
[Unclassified Title]

INTRODUCTION

In a recent NRL report (1), a method was devised which is capable of estimating a ship's course, speed, and target dimensions with a noncoherent side-looking satellite radar possessing two beams—one squinted forward and the other aft. Basically, the method uses the ship's projections on the two squinted beams for the estimations. Unfortunately, besides the correct estimate, three spurious pairs of estimates are given. However, this ambiguity is removed by using the target's change in range to select one of the four estimates.

Since this initial work, the estimation method has been modified, and the accuracy of the estimations has been determined by a Monte Carlo simulation.

BASIC ESTIMATION METHOD

The geometry of a two-look radar is shown in Fig. 1. One beam is squinted forward θ_1 radians off broadside and the other beam is squinted backward θ_2 radians. The lengths of the target projections along the two radar beams are shown in Figs. 2a and 2b to be

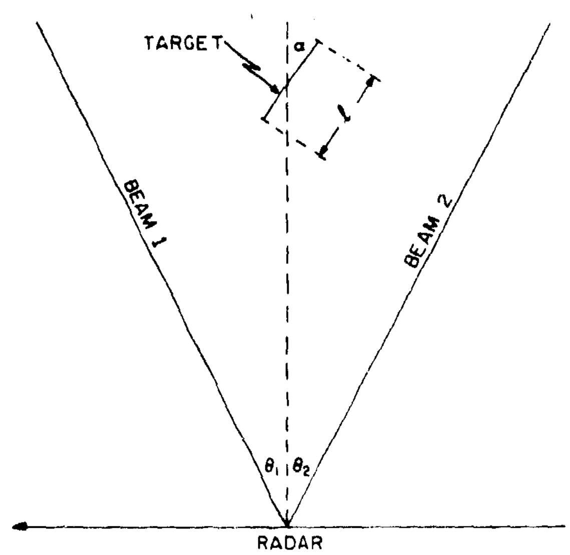


Fig. 1—Geometry of a two-look radar

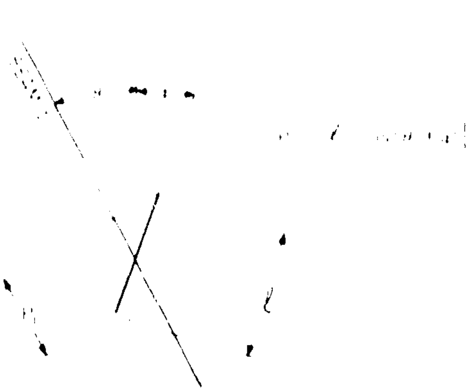


Fig. 2a Projection along beam 1

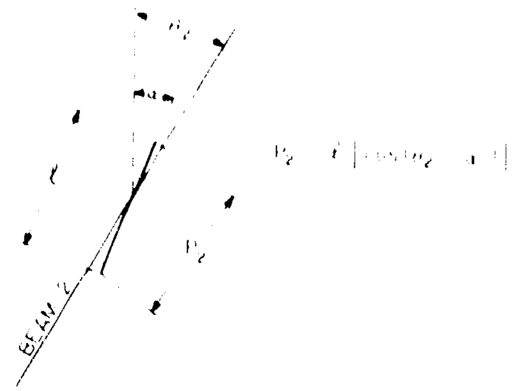


Fig. 2b Projection along beam 2

$$P_1 = \ell|\cos(\theta_1 + \alpha)| \qquad (1)$$

and

$$P_2 = \ell|\cos(\theta_2 - \alpha)|, \qquad (2)$$

where ℓ is the length of the target and α is its heading. Solving Eqs. (1) and (2) for α, one obtains

$$\alpha = \tan^{-1}\left[\frac{-P_1 \cos\theta_2 \pm P_2 \cos\theta_1}{P_1 \sin\theta_2 \pm P_2 \sin\theta_1}\right], \qquad (3)$$

an equation which has four solutions. The four solutions are shown in Fig. 3, two of the solutions being 180° inversions of the other solutions. To find the relationship between the two nontrivial solutions, consider the geometric construction shown in Fig. 4. If ℓ' is the estimated length of the second solution, then

$$\ell' = (P_1^2 + Z^2)^{1/2}, \qquad (4)$$

where

$$Z = (y + P_2)/\sin(\theta_1 + \theta_2) \qquad (5)$$

and

$$y = P_1 \cos(\theta_1 + \theta_2). \qquad (6)$$

Then, α' can be found by substituting ℓ' into Eq. (1), i.e.,

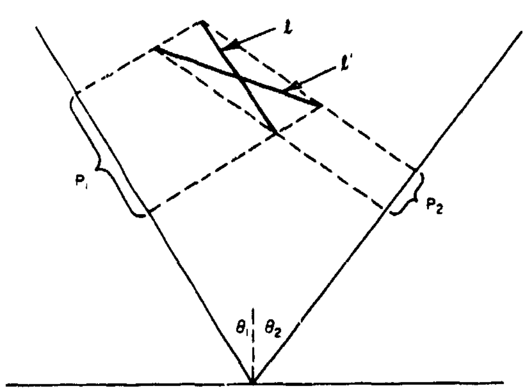


Fig. 3—Generation of the four solutions

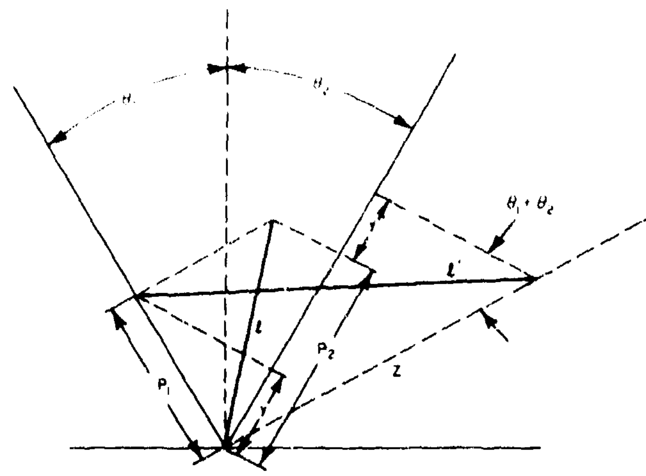


Fig. 4—Generation of the second solution from the first solution

$$\alpha' = \cos^{-1}(\pm P_1/\ell') - \theta_1. \tag{7}$$

As shown in Fig. 5, the two solutions* α and $\alpha' = F(\alpha)$ lie in the same quadrant on opposite sides of the dashed line perpendicular to the squinted beam. Moreover, $F(\alpha)$ has the following properties:

(a) $F(0) = \pi/2$

(b) $F(\pi/2 - \theta_1) = \pi/2 - \theta_1$

(c) $F(\alpha)$ is a monotonic decreasing function of α.

Statement (a) implies that when one solution is perpendicular to the radar's ground track, the other solution is parallel; statement (b) implies that both solutions are identical when $\alpha = \pi/2 - \theta_1$.

In Ref. 1, the correct solution was chosen by making use of the change in range ΔR, which is given in that report as

$$\Delta R = R_2 - R_1 = VT\cos\alpha/\cos\theta_2 + R_1(\cos\theta_1 - \cos\theta_2)/\cos\theta_2, \tag{8}$$

*The other two solutions are $\alpha + \pi$ and $\alpha' + \pi$.

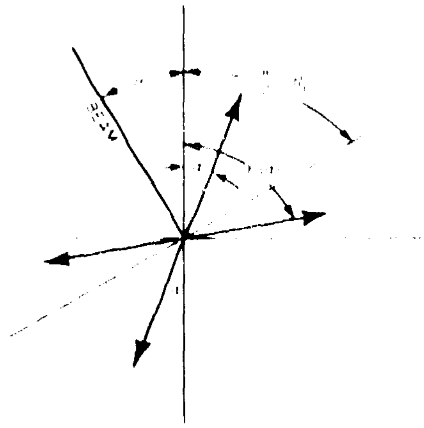


Fig. 5 Relationship between the two solutions α and F(α)

where V is the speed of the target, R_1 and R_2 are the measured target ranges, and T is the time between detections of the target in the two beams. The assumption underlying Eq. (8) is that the target is at the center of the beams, i.e., at the azimuths of θ_1 and θ_2. Of course, the target can be at positions within the beamwidth other than the center of the beam. Because of the narrow beamwidth, this azimuth error is very small, but conversion of this angular error to a linear error involves multiplication by the range to the target, causing position errors of the same order of magnitude as the change in range ΔR. To cope with this situation, a modification was made.

This modification involves estimating the target position in each beam, calculating an estimate of the target's velocity from the two positions, and using this velocity estimate to choose the correct heading. The linear errors of the position estimates* in the perpendicular and parallel directions are shown in Fig. 6 to be

$$D_e(\perp) = N\Delta D \cos\theta \sin\theta \qquad (9)\dagger$$

and

$$D_e(\parallel) = N\Delta D \cos^2\theta, \qquad (10)$$

where ΔD is the distance the radar moves between pulse transmissions and N is the error (in number of pulses) in locating the target within the beam.

If N_1 and N_2 are the errors for the two beam positions, the apparent distance components moved by the target (actual distance plus estimation error) are

*The estimation method is discussed in the next section.
†In this calculation and all following, it is assumed that $\theta_1 = \theta_2 = \theta$ with no loss of generality.

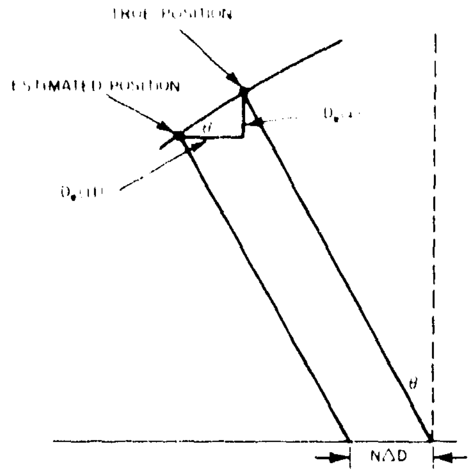


Fig. 6 Position estimation errors

$$D(\perp) = VT\cos\alpha + (-N_1 - N_2)\Delta D\cos\theta\sin\theta \tag{11}$$

and

$$D(||) = VT\sin\alpha + (N_2 - N_1)\Delta D\cos^2\theta; \tag{12}$$

the estimated speeds are

$$V(\perp) = D(\perp)/T \tag{13}$$

and

$$V(||) = D(||)/T. \tag{14}$$

The standard deviations of the speed errors are

$$\sigma(\perp) = \sqrt{2}\,\sigma_N\,\Delta D\cos\theta\sin\theta/T \tag{15}$$

and

$$\sigma(||) = \sqrt{2}\,\sigma_N\,\Delta D\cos^2\theta/T, \tag{16}$$

where σ_N is the standard deviation of the position error.

Since the position errors N_1 and N_2 are independent and since it has been shown that the probability density of the errors is Gaussian (2), the relative probability of the ith heading (i can take on four possible values) being correct is

$$p(\alpha_i) = \exp\left\{\frac{-[V(\perp) - \hat{V}\cos\alpha_i]^2}{2\sigma^2(\perp)}\right\}\exp\left\{\frac{-[V(||) - \hat{V}\sin\alpha_i]^2}{2\sigma^2(||)}\right\}, \tag{17}$$

where

$$V = [V^2(\mathrm{I}) + V^2(\mathrm{II})]^{1/2}. \tag{18}$$

There are two obvious estimates using these probabilities, the first being the maximum likelihood estimate. That is, choose the α_i that has the highest probability $p(\alpha_i)$. The second method is to weight the answers proportional to the probabilities, i.e.,

$$\hat{\alpha} = \tan^{-1}\left[\frac{\sum_{i=1}^{4} p(\alpha_i)\sin\alpha_i}{\sum_{i=1}^{4} p(\alpha_i)\cos\alpha_i}\right] \tag{19}$$

and

$$\hat{\ell} = \sum_{i=1}^{4} p(\alpha_i)\ell_i \Big/ \sum_{i=1}^{4} p(\alpha_i). \tag{20}$$

As will be shown later, these methods yield about the same results. However, at present, the best estimate is still not known.

POSITION ESTIMATES

Several estimation methods were investigated and the most accurate method involved threshold crossings of a moving window. The details of this method are as follows: Let P_{ij} be the ith returned pulse in the jth range cell. If the target is detected initially on the Ith pulse, the moving window threshold T_{mw} is defined by

$$T_{mw} \triangleq MW(i=I) = \sum_{k=0}^{K-1} S_{I+k}, \tag{21}$$

where K is the number of terms in the moving window, and

$$S_i = \sum_j P_{ij}, \tag{22}$$

where the j summation is over the range cells in which the target is present at the time of the initial detection. In terms of the notation of Ref. 1, the j summation goes from L_1 to L_2. Now define I' to be the largest i such that

$$MW(i) \geq T_{mw}. \tag{23}$$

Then, the estimate of the pulse at which the target is centered in the beam is

$$(I + I' + K)/2. \tag{24}$$

The accuracy of this method will depend on the size of the target. For the three basic targets that were considered:

<u>Tanker</u>

length ≈ 763 ft
width ≈ 102 ft
height ≈ 30 ft
cross section = 30,000 m^2

<u>Destroyer</u>

length = 450 ft
width = 50 ft
height = 18 ft
cross section = 10,000 m^2

<u>Trawler</u>

length ≈ 150 ft
width = 30 ft
height = 10 ft
cross section = 2000 m^2

the standard deviations of the estimate given by Eq. (24) were 4.8, 10.7, and 23.6 pulses respectively for the tanker, destroyer, and trawler, K being equal to 20.

Some of the other estimation methods tried were beam splitting techniques and cross correlation of the returned signal with the antenna pattern. Neither of these methods is as good as the threshold crossing method using the moving window.

MONTE CARLO RESULTS

The basic radar parameters that were used in the simulation are

radar wavelength = 0.79965 ft
prf = 78 pps
beamwidth = 0.63°
squint angle = 34.4° (0.6 radians)
range resolution = 50 ft
range = 700 naut mi
radar altitude = 200 naut mi

In the simulation, the projections P_1 and P_2 were estimated by using Method I*; the parameter I_M was set equal to 50. The ship's speed V was chosen to be uniformly distributed between 15 and 25 knots, and the ship's heading was chosen to be uniformly distributed in a 6° interval that was centered at one of the following values: 0°, 17°, 34°, 63°, and 90°. For each of the five basic headings, ten cases were run. The errors for the 50 cases are given in Table 1 for the tanker, Table 2 for the destroyer, and Table 3 for the trawler; a summary of the means and standard deviations is given in Table 4. It should be noted that the method labeled "correct estimate" in Table 4 is *not* obtainable. When the position estimates have large errors, one of the spurious estimates may be chosen instead of the correct estimate. This condition is reflected by the two realizable methods which have larger errors than the "correct estimate." The following conclusions are drawn from Table 4:

1. There is very little difference between the probability weighting method and the maximum likelihood method.

2. The estimations of length are fairly good; the root-mean-squared error $[\sqrt{E(L^2)}]$ is about 50 ft for each ship.

3. The standard deviations of the heading error are about 50, 200, and 400 percent greater than the "correct estimate" for the tanker, destroyer, and trawler respectively. The large errors are caused by the fact that for the smaller ships the position errors are rather large and the correct estimate (out of the four possible estimates) is not chosen.

4. No useful speed informatoni is obtained on the destroyer or trawler. That is, the standard deviation of the speed estimate is greater than that of the underlying population which was 2.9 knots.

ESTIMATION METHOD USING SHIP'S WIDTH

If Tables 1 through 3 are considered, under the heading "correct estimate," one notices a definite correlation between the errors and the heading angle. Specifically for heading angles centered around 0°, 17°, and 34°, the length and heading are underestimated; for heading angles centered around 63° and 90°, the length and heading are overestimated. In an attempt to remedy this situation, the projections given by Eqs. (1) and (2) were modified to include the width of the ship. As suggested in Ref. 1, the projections are given by

$$P_1 = \ell|\cos(\theta_1 + \alpha)| + W \sin^2(\theta_1 + \alpha) \tag{25}$$

and

$$P_2 = \ell|\cos(\theta_2 - \alpha)| + W \sin^2(\theta_2 - \alpha); \tag{26}$$

*Two methods for estimating the ship's projections are discussed in Appendix A.

Table 1
Estimation errors for the Tanker

True Heading (deg.)	Speed Error (knots)	Probability Weighting		Maximum Likelihood		Correct Estimate	
		Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
− 2.1	2.5	− 26	1.1	− 26	1.1	26	1.1
2.1	6.3	− 12	1.7	− 12	1.7	12	1.7
− 0.3	2.3	− 13	0.6	− 13	0.6	− 13	0.6
− 2.7	− 4.0	− 64	2.9	− 65	2.9	− 65	2.9
0.6	− 1.1	− 24	1.1	− 24	1.1	− 24	1.1
− 1.9	− 0.4	− 13	− 0.6	− 13	− 0.6	− 13	− 0.6
1.4	− 0.4	− 14	− 1.7	− 14	− 1.7	− 14	− 1.7
− 1.0	− 0.9	− 28	1.7	28	1.7	− 28	1.7
2.3	− 1.4	− 22	1.1	− 22	1.1	− 22	1.1
− 2.5	0.1	1	1.7	1	1.7	1	1.7
16.4	− 1.8	− 25	− 1.7	− 26	− 2.3	− 26	− 2.3
18.0	2.3	− 3	− 6.3	− 3	− 6.3	− 3	− 6.3
19.6	− 1.6	− 34	− 5.7	− 34	− 5.7	− 34	− 5.7
15.6	− 2.9	− 26	− 0.6	− 26	− 0.6	− 26	− 0.6
17.2	2.0	− 42	− 0.6	− 42	− 0.6	− 42	− 0.6
18.9	5.0	− 19	0.6	− 19	0.6	− 19	0.6
14.8	− 1.8	− 97	− 2.9	− 97	− 2.9	− 97	− 2.9
16.4	3.5	2	− 1.1	2	− 1.1	3	− 1.1
18.1	− 6.6	− 11	− 5.7	− 12	− 5.7	− 12	− 5.7
19.7	0.6	− 41	4.0	− 46	3.4	− 46	3.4
32.8	2.8	− 57	− 4.6	− 58	− 4.6	− 58	− 4.6
34.5	4.0	− 43	0.6	− 65	− 3.4	− 65	− 3.4
36.1	− 4.5	0	10.3	− 52	0.0	− 52	0.0
32.0	− 6.4	8	− 1.1	− 18	− 5.7	− 18	− 5.7
33.7	4.7	122	21.2	230	41.8	− 7	− 4.0
33.0	− 0.6	2	− 1.7	− 16	− 4.6	− 16	− 4.6
36.7	− 6.0	− 35	− 9.2	− 35	− 9.2	− 35	− 9.2
34.7	− 4.4	− 23	− 4.0	− 27	− 4.6	− 27	− 4.6
32.6	2.8	− 19	− 1.1	− 27	− 2.9	− 27	− 2.9
36.3	7.2	− 47	− 4.6	− 49	− 5.2	− 49	− 5.2
62.9	− 4.2	105	6.3	126	10.9	126	10.9
60.9	9.9	− 1	− 1.1	− 39	6.9	− 39	6.9
64.6	8.5	55	− 4.0	62	5.2	62	5.2
62.5	− 1.7	− 87	− 2.9	− 37	9.7	− 37	9.7
60.5	4.3	76	6.3	85	8.0	85	8.0
64.2	0.7	91	6.3	96	7.4	96	7.4
62.1	6.8	54	4.6	59	5.7	59	5.7
65.8	− 0.6	− 26	8.0	− 25	8.0	− 25	8.0
63.8	1.7	−121	−28.1	−126	28.6	40	8.0
61.7	− 0.2	38	5.2	44	6.3	44	6.3
92.3	9.2	90	0.6	90	0.6	90	0.6
90.3	− 6.7	16	0.6	19	0.0	19	0.0
88.3	0.5	24	− 1.7	24	− 1.7	24	− 1.7
91.9	6.2	71	2.9	71	2.9	71	2.9
89.9	− 5.8	− 29	− 6.9	1	− 0.6	1	− 0.6
87.8	− 7.3	78	1.7	78	1.7	78	1.7
91.5	− 1.6	61	0.0	61	0.0	61	0.0
89.5	12.6	89	1.1	89	1.1	89	1.1
87.5	7.5	47	− 1.1	47	− 1.1	47	− 1.1
91.2	3.6	64	− 2.9	64	− 2.9	64	− 2.9

Table 2
Estimation Errors for the Destroyer

True Heading (deg.)	Speed Error (knots)	Probability Weighting		Maximum Likelihood		Correct Estimate	
		Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
2.7	7.1	74	39.5	20	4.0	20	4.0
−2.2	5.4	32	1.1	32	1.1	32	1.1
−0.8	7.0	11	1.1	−11	1.1	−11	1.1
−2.8	0.7	5	4.0	5	4.0	−5	4.0
0.6	−7.4	−11	2.3	−14	1.1	−14	1.1
2.3	−4.3	37	1.7	31	−2.9	31	−2.9
2.4	−0.3	−16	1.7	−16	1.7	−16	1.7
1.9	−1.1	−25	6.9	−25	6.9	−25	6.9
−2.7	0.1	1	2.9	1	2.9	1	2.9
0.0	−1.0	−7	1.1	−7	1.1	−7	1.1
19.9	−2.1	−13	5.7	−18	4.0	−18	4.0
15.0	0.8	51	13.8	−6	−5.7	−6	−5.7
16.4	10.3	−16	−6.9	−16	−6.9	−16	−6.9
14.4	−5.5	−13	−8.6	−37	−2.9	−37	−2.9
17.8	10.8	−11	−1.7	−11	−1.7	−11	−1.7
19.5	5.4	−1	−6.3	−1	−6.3	−1	−6.3
19.6	2.8	−12	0.0	−12	0.0	−12	0.0
19.1	4.9	−17	−1.1	−17	−1.1	−17	−1.1
14.4	−1.4	−26	−5.7	−27	−5.7	−27	−5.7
16.0	11.4	−5	0.6	−5	0.6	−5	0.6
34.7	1.4	1	15.5	−55	−5.2	−55	−5.2
33.2	−10.6	3	−1.7	−6	−4.0	−6	−4.0
35.8	−0.6	35	8.0	9	0.6	9	0.6
36.4	13.4	−23	−4.0	−23	−4.0	−23	−4.0
33.4	−4.3	−6	−1.7	−12	−3.4	−12	−3.4
34.8	35.1	20	−3.4	18	−4.0	18	−4.0
36.7	−1.1	20	15.5	−72	−3.4	−72	−3.4
34.3	4.5	12	−1.7	11	−1.7	11	−1.7
33.1	0.6	−42	−4.6	−42	−4.6	−42	−4.6
34.8	10.1	−5	−8.6	−5	−8.6	−5	−8.6
62.6	−2.6	−12	−5.2	24	8.0	24	8.0
65.8	15.0	48	5.7	55	8.0	55	8.0
63.5	9.9	−49	0.6	−31	−8.6	−31	−8.6
64.5	−1.3	−60	8.0	−60	8.0	−60	8.0
65.8	−1.0	39	3.4	46	6.3	46	6.3
64.9	12.8	77	6.3	77	6.3	77	6.3
63.5	5.9	45	11.5	45	11.5	45	11.5
62.9	−4.9	74	0.6	91	5.7	91	5.7
65.5	0.6	−78	−28.6	−80	29.2	11	5.7
64.0	13.9	63	4.6	67	5.7	67	5.7
90.4	1.1	70	3.4	70	3.4	70	3.4
90.8	20.9	44	−0.6	44	−0.6	44	−0.6
88.4	6.1	71	−0.6	71	−0.6	71	−0.6
88.0	−8.1	−77	−73.9	−123	−81.9	26	−1.1
88.0	−5.1	−1	0.6	−1	0.6	−1	0.6
88.4	2.2	92	−2.9	92	−2.9	92	−2.9
89.5	14.8	53	−2.3	53	−2.3	53	−2.3
90.7	4.5	51	−1.1	51	−1.1	51	−1.1
89.6	−7.8	17	−1.1	17	−1.1	17	−1.1
90.2	18.7	33	−4.6	33	−4.6	33	−4.6

Table 3
Estimation Errors for the Trawler

True Heading (deg.)	Speed Error (knots)	Probability Weighting		Maximum Likelihood		Correct Estimate	
		Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
− 2.1	−15.2	71	−121.5	108	84.2	28	5.7
2.1	3.1	1	− 4.6	1	− 4.6	1	− 4.6
− 0.3	0.9	86	− 76.8	92	− 81.4	22	−16.6
− 2.7	22.2	14	− 12.0	14	− 12.0	14	−12.0
0.6	13.9	113	− 89.4	113	89.4	30	0.6
− 1.9	43.2	58	− 1.1	58	− 1.1	58	− 1.1
1.4	− 1.8	27	− 2.3	27	− 2.3	27	− 2.3
− 1.0	1.7	71	80.2	77	85.4	8	12.6
2.3	− 2.8	− 20	3.4	− 22	1.1	−22	1.1
− 2.5	30.8	118	97.4	118	97.4	36	− 8.0
16.4	41.3	56	7.4	38	− 6.9	38	− 6.9
18.0	− 3.6	22	3.4	21	2.9	21	2.9
19.6	18.5	64	67.6	64	67.6	− 3	−14.3
15.6	11.9	20	175.3	20	175.3	20	− 4.6
17.2	− 6.3	37	49.3	56	69.9	− 8	−10.9
18.9	4.2	− 7	− 32.7	− 7	− 32.7	− 7	−32.7
14.8	6.1	89	67.6	89	67.6	18	0.6
16.4	− 7.5	3	−145.5	26	−117.5	−21	6.9
18.1	5.1	23	12.0	23	11.5	23	11.5
19.7	− 8.2	23	6.3	5	− 9.7	5	− 9.7
32.8	11.9	102	42.4	102	42.4	41	− 3.4
34.5	0.3	− 16	− 17.2	− 16	− 17.2	−16	−17.2
36.1	8.1	− 17	− 9.7	− 17	− 9.7	−17	− 9.7
32.0	4.4	− 11	− 18.9	− 11	− 18.9	−11	−18.9
33.7	2.4	59	− 4.0	59	− 4.0	59	− 4.0
33.0	− 6.8	55	24.6	82	50.4	13	−18.3
36.7	− 3.0	− 0	− 14.9	− 0	− 14.9	− 6	−14.9
34.7	2.6	10	− 5.7	10	− 5.7	10	− 5.7
32.6	8.3	19	− 9.7	19	− 9.7	19	− 9.7
36.3	9.3	− 9	− 2.9	− 23	− 19.5	−23	−19.5
62.9	2.1	− 7	− 37.2	− 7	− 37.2	44	14.3
60.9	− 5.5	11	24.6	11	24.6	11	24.6
64.6	−13.5	26	154.1	7	136.4	68	15.5
62.5	7.0	− 3	− 49.3	− 3	− 49.3	61	21.2
60.5	− 7.6	27	6.9	31	10.9	31	10.9
64.2	40.4	88	14.9	88	14.9	88	14.9
62.1	59.8	56	17.8	56	17.8	56	17.8
65.8	31.1	40	16.6	40	16.6	40	16.6
63.8	8.2	39	− 4.0	58	13.8	58	13.8
61.7	20.1	61	11.5	61	11.5	61	11.5
92.3	− 3.8	99	− 3.4	99	− 3.4	99	− 3.4
90.3	− 4.0	− 16	− 73.3	− 31	− 94.0	24	1.7
88.3	− 1.1	14	13.2	14	13.2	14	13.2
91.9	−10.2	66	− 9.2	68	− 9.2	68	− 9.2
89.9	− 0	− 33	94.5	− 35	96.8	18	− 2.9
87.8	5.1	16	−176.5	16	−176.5	16	3.4
91.5	5.5	14	− 2.3	14	− 2.3	14	− 2.3
89.5	17.3	53	6.9	53	6.9	53	6.9
87.5	3.2	11	− 13.8	11	− 13.8	11	−13.8
91.2	22.0	38	− 10.3	39	− 8.6	39	− 8.6

Table 4
Summary of Estimation Errors

Ship Type	Decision Process	E(L)	$\sqrt{E(L^2)}$	σ(L)	E(H)	$\sqrt{E(H^2)}$	σ(H)	E(S)	σ(S)
Tanker	Probability Weighting	2	53	53	0.1	6.3	6.3	0.8	4.7
	Maximum Likelihood	3	62	62	0.5	8.4	8.4	0.8	4.7
	Correct Estimate	2	50	50	0.4	4.5	4.5	0.8	4.7
Destroyer	Probability Weighting	10	41	40	0.3	13.7	13.7	3.4	8.6
	Maximum Likelihood	4	44	44	1.9	13.1	13.0	3.4	8.6
	Correct Estimate	8	39	38	0.4	4.7	4.6	3.4	8.6
Trawler	Probability Weighting	33	51	37	4.6	61.4	61.2	7.4	15.4
	Maximum Likelihood	34	53	41	5.6	59.8	59.5	7.4	15.4
	Correct Estimate	25	38	28	−1.4	12.4	12.3	7.4	15.4

Legend:
E ≡ Expected value.
σ ≡ Standard deviation.
L ≡ Length error (feet).
H ≡ Heading error (degrees).
S ≡ Speed error (knots).

the width W is related to the length ℓ by

$$W = 24.88 + 0.000128\ell^2, \tag{27}$$

an equation empirically derived from actual ship dimensions. These three equations are solved simultaneously to obtain the heading, length, and width. To find the accuracy of this new method, the simulation was repeated for the tanker, destroyer, and trawler; the new results appear in Tables 5, 6, and 7 respectively. Histograms for the various errors are plotted in Fig. 7 thru 12, and the results are summarized in Table 8. Comparing the method using the ship's width (Table 8) with the method that does not (Table 4), one can draw the following conclusions:

1. The heading errors are about the same for both methods;

2. The length errors are about 25 percent smaller for the new method which includes the width;

3. Again, no useful speed information is obtained.

The estimates of the ship's length are very good. The standard deviation of the length error is about 30 ft, and over 90 percent of the estimates are within 50 ft of the true length, 50 ft being the range resolution of the radar system. These results can be further improved

Table 5
Estimation Errors for the Tanker

True Heading (deg.)	Probability Weighting		Maximum Likelihood		Correct Estimate	
	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
− 2.1	− 60	0.9	61	0.8	61	0.8
2.1	44	3.4	47	2.8	47	2.8
− 0.3	− 48	0.9	48	0.9	48	0.9
− 2.7	95	3.2	96	3.3	96	3.3
0.6	− 58	1.7	58	1.7	− 58	1.7
− 1.9	− 49	− 1.1	49	1.1	49	1.1
1.4	47	− 1.9	− 49	1.6	− 49	−1.6
− 1.0	− 62	1.8	− 62	1.8	− 62	1.8
2.3	− 48	2.1	− 56	2.4	− 56	2.4
− 2.5	− 35	1.1	− 35	1.2	− 35	1.2
16.4	− 47	2.9	− 50	2.0	− 50	2.0
18.0	− 31	− 2.4	− 32	− 2.6	− 32	−2.6
19.6	− 58	− 1.7	− 59	− 1.8	− 59	−1.8
15.6	− 49	4.2	− 50	4.1	− 50	4.1
17.2	− 63	4.5	− 63	4.4	− 63	4.4
18.9	− 36	6.2	− 38	5.7	− 38	5.7
14.8	− 24	3.3	− 25	2.9	− 25	2.9
16.4	−120	1.3	−120	1.1	−120	1.1
18.1	− 32	0.3	− 41	− 2.2	− 41	−2.2
19.7	− 52	11.1	− 58	9.4	− 58	9.4
32.8	− 42	7.6	− 62	2.1	− 62	2.1
34.5	− 39	10.7	− 64	3.6	− 64	3.6
36.1	− 20	14.1	− 45	7.4	− 45	7.4
32.0	− 6	6.1	− 25	1.0	− 25	1.0
33.7	24	12.1	− 9	3.4	− 9	3.4
33.0	10	10.1	− 20	2.1	− 20	2.1
36.7	− 34	− 0.5	− 40	− 2.2	− 40	−2.2
34.7	− 16	5.8	− 29	2.2	− 29	2.2
32.6	− 3	11.3	− 29	4.2	− 29	4.2
36.3	− 36	5.7	− 50	1.9	− 50	1.9
62.9	7	− 1.0	29	5.2	29	5.2
60.9	− 56	− 6.1	− 79	−12.5	− 32	0.8
64.6	− 43	− 8.2	− 17	− 0.6	− 17	−0.6
62.5	−146	− 8.7	−184	−21.2	−108	4.5
60.5	− 19	− 5.2	7	1.9	7	1.9
64.2	− 20	− 6.7	9	1.7	9	1.7
62.1	− 35	− 6.3	− 14	− 0.3	− 14	−0.3
65.8	−112	0.1	−102	3.4	−102	3.4
63.8	− 8	− 9.1	− 41	2.9	− 41	2.9
61.7	− 38	− 2.5	− 28	0.3	− 28	0.3
92.3	− 23	2.0	− 23	1.9	− 23	1.9
90.3	−106	− 0.2	− 82	0.4	− 82	0.4
88.3	− 78	− 3.1	− 77	− 3.0	− 77	−3.0
91.9	− 38	4.8	− 37	4.5	− 37	4.5
89.9	−138	− 9.7	− 99	− 0.5	− 99	−0.5
87.8	− 34	1.4	− 34	1.4	− 34	1.4
91.5	− 49	1.0	− 48	0.7	− 48	0.7
89.5	− 24	1.7	− 24	1.7	− 24	1.7
87.5	− 58	− 2.2	− 58	− 2.2	− 58	−2.2
91.2	− 45	− 3.2	− 45	− 3.2	− 45	−3.2

Note: Speed errors are the same as those given in Table 1.

Table 6
Estimation Errors for the Destroyer

True Heading (deg.)	Probability Weighting		Maximum Likelihood		Correct Estimate	
	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
2.7	5	20.0	36	5.6	36	5.6
2.2	12	0.9	12	0.9	12	0.9
− 0.8	30	0.9	30	0.9	−30	0.9
− 2.8	22	3.4	− 24	4.2	−24	4.2
0.6	− 29	3.5	− 32	2.1	−32	2.1
2.3	17	− 3.9	12	− 3.3	12	−3.3
2.4	− 34	2.5	− 34	2.6	−34	2.6
1.9	− 42	8.9	− 42	8.9	−42	8.9
− 2.7	− 18	3.1	− 18	3.1	−18	3.1
− 1.1	4	− 4.9	4	− 4.9	4	−4.9
19.9	− 20	11.3	− 24	9.7	−24	9.7
15.0	10	11.3	− 21	− 3.1	−21	−3.1
16.4	− 32	− 4.4	− 32	− 4.4	−32	−4.4
14.4	− 8	− 8.6	− 50	0.2	−50	0.2
17.8	− 23	2.5	− 23	2.5	−23	2.5
19.5	− 15	− 2.5	− 15	− 2.6	−15	−2.6
19.6	− 21	5.2	− 22	4.7	−22	4.7
19.1	− 26	3.6	− 27	3.5	−27	3.5
14.4	− 41	− 3.1	− 43	− 3.7	−43	−3.7
16.0	− 16	4.9	− 17	4.5	−17	4.5
34.7	− 21	18.1	− 57	1.2	−57	1.2
33.2	6	7.6	− 7	2.0	− 7	2.0
35.8	37	16.9	14	7.0	14	7.0
36.4	− 21	3.1	− 23	2.3	−23	2.3
33.4	1	8.9	− 13	2.7	−13	2.7
34.8	21	2.9	19	2.2	19	2.2
36.7	− 48	14.0	− 70	3.2	−70	3.2
34.3	22	8.5	11	4.4	11	4.4
33.1	− 43	2.3	− 44	1.6	−44	1.6
34.8	− 5	− 1.0	− 11	− 3.2	−11	−3.2
62.6	− 38	− 6.8	− 18	3.1	−18	3.1
65.8	− 28	−12.6	− 60	−27.7	6	3.9
63.5	− 89	− 7.4	− 70	3.5	−70	3.5
64.5	−105	− 0.8	− 97	3.6	−97	3.6
65.8	− 21	− 8.9	1	1.3	1	1.3
64.9	24	− 1.9	31	1.2	31	1.2
63.5	− 10	4.4	− 4	7.2	− 4	7.2
62.9	34	− 4.3	46	0.5	46	0.5
65.5	− 67	−17.6	− 77	−22.6	−30	0.7
64.0	2	− 8.6	23	0.5	23	0.5
90.4	9	5.0	9	5.0	9	5.0
90.8	− 15	− 0.8	− 14	− 0.8	−14	−0.8
88.4	9	− 1.3	9	− 1.3	9	−1.3
88.0	−119	−86.9	−136	−79.5	−30	−1.8
88.0	− 54	0.2	− 54	0.2	−54	0.2
88.4	28	− 4.0	28	− 4.0	28	−4.0
89.5	− 6	− 3.3	− 6	− 3.3	− 6	−3.3
90.7	− 8	− 1.4	− 8	− 1.5	− 8	−1.5
89.6	− 39	− 1.6	− 39	− 1.8	−39	−1.8
90.2	− 23	− 6.0	− 23	− 6.0	−23	−6.0

Note: Speed errors are the same as those given in Table 2.

Table 7
Estimation Errors for the Trawler

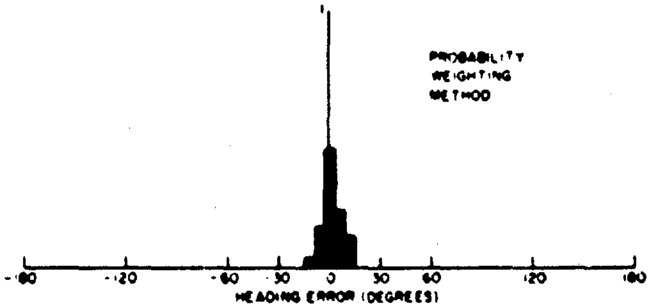
True Heading (deg.)	Probability Weighting		Maximum Likelihood		Correct Estimate	
	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
−2.1	51	−75.3	70	82.7	18	−9.6
2.1	−10	5.7	−10	−5.7	10	−5.7
−0.3	51	−69.2	58	−78.2	15	−23.8
−2.7	6	−18.7	6	−18.7	6	−18.7
0.6	73	88.2	74	89.5	19	−0.8
−1.9	47	−2.5	47	−2.5	47	−2.5
1.4	17	−1.9	16	−2.1	16	−2.1
−1.0	19	48.1	−1	18.7	−1	18.7
2.3	−29	9.3	−33	3.6	−33	3.6
−2.5	81	99.0	81	99.0	27	−12.8
16.4	29	−2.3	29	−2.6	29	−2.6
18.0	20	13.9	17	10.5	17	10.5
19.6	30	66.3	30	66.3	−12	−10.6
15.6	11	−179.7	11	−179.7	11	0.3
17.2	2	15.1	−18	−7.5	−18	−7.5
18.9	−12	−40.8	−13	−40.3	−13	−40.3
14.8	34	38.6	54	65.1	11	7.6
16.4	−10	−135.3	−3	−123.2	−24	18.0
18.1	25	24.3	22	20.7	22	20.7
19.7	13	11.0	−4	−5.0	−4	−5.0
32.8	72	36.8	72	37.2	42	5.2
34.5	−20	−6.3	−21	−7.9	−21	−7.9
36.1	−18	1.6	−18	1.6	−18	1.6
32.0	−18	−12.1	−18	−12.2	−18	−12.2
33.7	58	4.4	58	4.3	58	4.3
33.0	26	12.4	5	−11.6	5	−11.6
36.7	−4	−5.4	−4	−5.9	−4	−5.9
34.7	11	7.0	9	4.3	9	4.3
32.6	15	−1.1	15	−1.2	15	−1.2
36.3	−25	−1.7	−30	−10.4	−30	−10.4
62.9	−8	−26.2	−8	−26.3	15	8.9
60.9	−22	20.7	−21	21.9	−21	21.9
64.6	23	−4.6	36	11.2	36	11.2
62.5	−10	−41.9	−10	−42.3	28	18.4
60.5	4	−1.9	7	3.4	7	3.4
64.2	57	10.8	57	10.8	57	10.8
62.1	25	12.3	25	13.0	25	13.0
65.8	8	12.1	8	12.1	8	12.1
63.8	15	−10.1	29	8.8	29	8.8
61.7	34	4.9	34	5.0	34	5.0
92.3	61	−5.3	62	−4.3	62	−4.3
90.3	−25	−93.4	−41	−96.9	−10	2.1
88.3	−17	24.9	−14	19.1	−14	19.1
91.9	22	−25.0	34	−11.7	34	−11.7
89.9	−43	100.4	−43	101.4	−16	−4.6
87.8	−17	−175.8	−17	−175.8	−17	4.2
91.5	−18	−3.3	−18	−3.3	−18	−3.3
89.5	20	9.8	20	9.8	20	9.8
87.5	−13	−22.7	−13	−22.3	−13	−22.3
91.2	−5	−33.6	8	−12.4	8	−12.4

Note: Speed errors are the same as those given in Table 3.

Table 8
Summary of Estimation Errors

Ship Fype	Decision Process	E(L)	$\sqrt{E(L^2)}$	$\sigma(L)$	E(H)	$\sqrt{E(H^2)}$	$\sigma(H)$	E(S)	$\sigma(S)$
Tanker	Probability Weighting	47	58	34	1.3	5.7	5.6	0.8	4.7
	Maximum Likelihood	48	59	34	0.9	4.6	4.5	0.8	4.7
	Correct Estimate	46	54	29	1.6	3.4	2.6	0.8	4.7
Destroyer	Probability Weighting	18	37	32	0.6	14.5	14.4	3.4	8.6
	Maximum Likelihood	22	40	33	1.4	12.9	12.8	3.4	8.6
	Correct Estimate	18	33	28	1.3	3.8	3.5	3.4	8.6
Trawler	Probability Weighting	13	32	29	6.6	54.4	54.0	7.4	15.4
	Maximum Likelihood	13	34	32	7.1	54.3	53.8	7.4	15.4
	Correct Estimate	8	25	24	0.4	12.5	12.5	7.4	15.4

Legend:
- E Expected value.
- σ Standard deviation.
- L Length error (feet).
- H Heading error (degrees).
- S Speed error (knots).



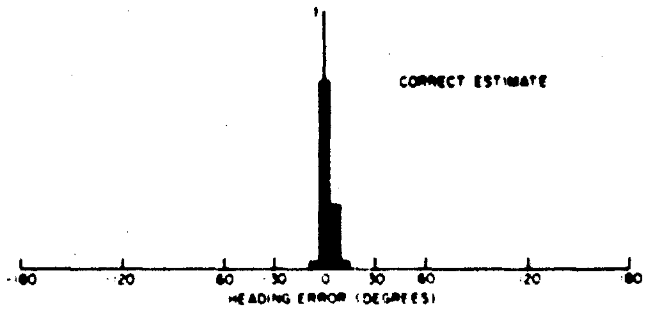


Fig. 7 Relative frequency of the heading error for the tanker

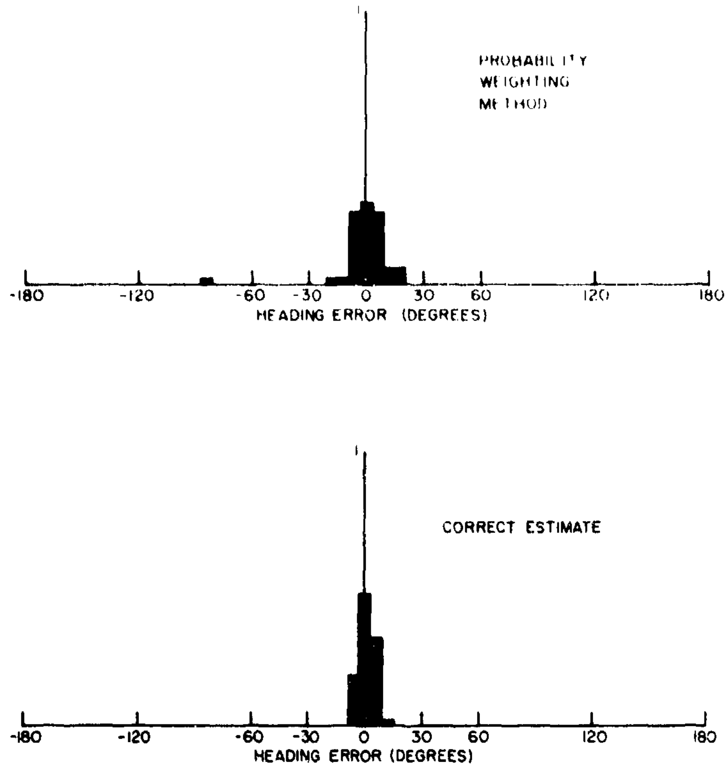


Fig. 8—Relative frequency of the heading error for the destroyer

by decreasing the pulse width of the radar. The results for 25-ft range resolution are shown* in Tables 9 and 10. The standard deviation of the length error was reduced from 30 to 20 ft.

SUMMARY

A method has been devised which is capable of estimating a ship's heading and length with a noncoherent side-looking radar possessing two beams—one squinted forward and the other aft. This method uses the ship's projections on the two squinted beams for the estimation. Unfortunately, besides the correct estimate, three spurious pairs of estimates are given. This ambiguity is removed by estimating the target's position in each squinted beam and then using the target's estimated velocity, which is derived from the two positions, to select one of the four estimates.

*The radar system is designed to detect a 200-square meter nonfluctuating target at a detection probability of 0.9 and a false alarm probability of 10^{-10}. Thus, the only effect of reducing the pulse width is to decrease the signal-to-noise ratio in a target range cell by 3 dB.

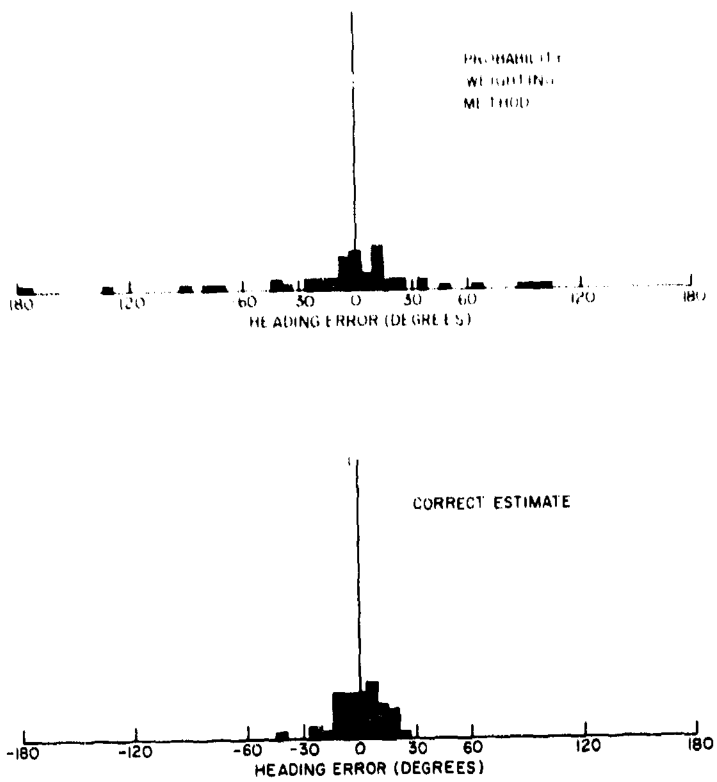


Fig. 9—Relative frequency of the heading error for the trawler

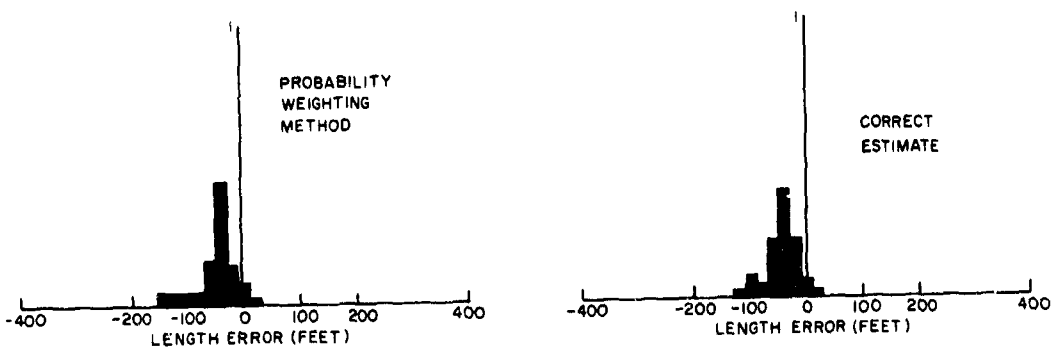


Fig. 10—Relative frequency of the length error for the tanker

Table 9
Estimation Errors for the Trawler

True Heading (deg.)	Probability Weighting		Maximum Likelihood		Correct Estimate	
	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)	Length (ft)	Heading (deg.)
2.1	12	98.9	13	98.4	19	24.4
2.1	7	-14.5	4	5.2	4	5.2
0.3	26	8.1	26	8.1	26	8.1
-2.7	-14	-7.3	-14	-7.3	-14	7.3
0.6	-26	-47.0	-33	-37.2	-33	37.2
1.9	13	-18.3	13	-18.3	-13	-18.3
1.4	-11	-15.3	-11	-15.2	-11	-15.2
-1.0	-31	41.2	-37	36.7	-37	36.7
2.3	-28	0.5	-30	1.0	-30	1.0
-2.5	4	100.2	7	105.0	-22	-24.1
16.4	-13	0.0	-23	-7.8	-23	-7.8
18.0	-3	48.4	7	67.6	-28	-7.2
19.6	-31	20.3	-32	18.8	-32	18.8
15.6	-19	-0.7	-22	-3.2	-22	-3.2
17.2	-33	-1.0	-33	-1.0	-33	-1.0
18.9	9	11.9	9	11.8	9	11.8
14.8	-26	13.6	-29	14.1	-29	14.1
16.4	-23	-14.2	-34	15.2	-34	15.2
18.1	-25	52.3	-17	77.5	-46	-30.8
19.7	-5	13.3	-13	0.5	-13	0.5
32.8	37	36.5	40	40.4	10	0.7
34.5	-18	-3.8	-22	-8.9	-22	-8.9
36.1	-23	17.0	-26	10.6	-26	10.6
32.0	-25	1.7	-25	1.5	-25	1.5
33.7	-14	3.1	-14	2.9	-14	2.9
33.0	0	19.8	-9	6.2	-9	6.2
36.7	-10	7.6	-13	4.0	-13	4.0
34.7	7	4.4	7	4.0	7	4.0
32.6	5	4.6	5	4.5	5	4.5
36.3	8	31.9	15	42.9	-17	-13.4
62.9	-22	-22.6	-22	-22.6	-6	6.1
60.9	-51	6.5	-50	9.5	-50	9.5
64.6	-6	132.5	-6	132.4	35	18.1
62.5	-13	-6.3	-10	-1.6	-10	-1.6
60.5	-27	20.6	-26	21.4	-26	21.4
64.2	-50	23.8	-50	23.8	-50	23.8
62.1	-58	-41.4	-60	-49.1	-50	16.1
65.8	-8	2.9	-8	2.9	-8	2.9
63.8	-8	-21.8	-13	-29.9	11	9.3
61.7	-1	-3.6	3	2.4	3	2.4
92.3	5	1.9	5	1.9	5	1.9
90.3	-50	6.8	-50	6.8	-50	6.8
88.3	-11	-14.4	-10	-13.3	-10	-13.3
91.9	-35	-1.2	-35	-1.4	-35	-1.4
89.9	-34	174.4	-34	174.3	-34	-5.7
87.8	-1	8.0	-0	7.1	-0	7.1
91.5	-22	-4.6	-22	-4.6	-22	-4.6
89.5	-25	-1.3	-23	3.8	-23	3.8
87.5	-5	5.3	-2	0.7	-2	0.7
91.2	-38	-7.9	-38	-7.9	-38	-7.9

Note: Speed errors are the same as those given in Table 3.

Table 10
Summary of Estimation Errors

Ship Type	Decision Process	E(L)	$\sqrt{E(L^2)}$	$\sigma(L)$	E(H)	$\sqrt{E(H^2)}$	$\sigma(H)$	E(S)	$\sigma(S)$
Trawler	Probability Weighting	-16	24	18	9.1	41.5	40.5	7.4	15.4
	Maximum Likelihood	-17	25	19	10.2	43.1	41.8	7.4	[illegible]
	Correct Estimate	-19	26	18	1.3	13.7	13.6	7.4	15.4

Legend:
E = Expected value.
σ = Standard deviation.
L = Length error (feet).
H = Heading error (degrees).
S = Speed error (knots).

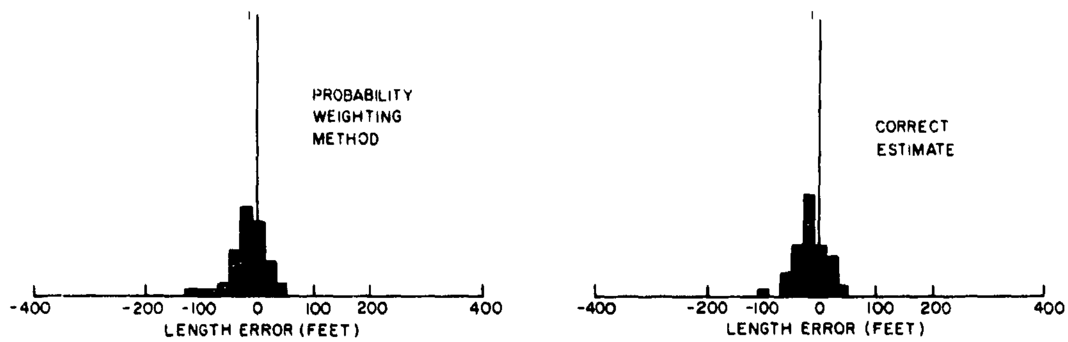


Fig. 11—Relative frequency of the length error for the destroyer

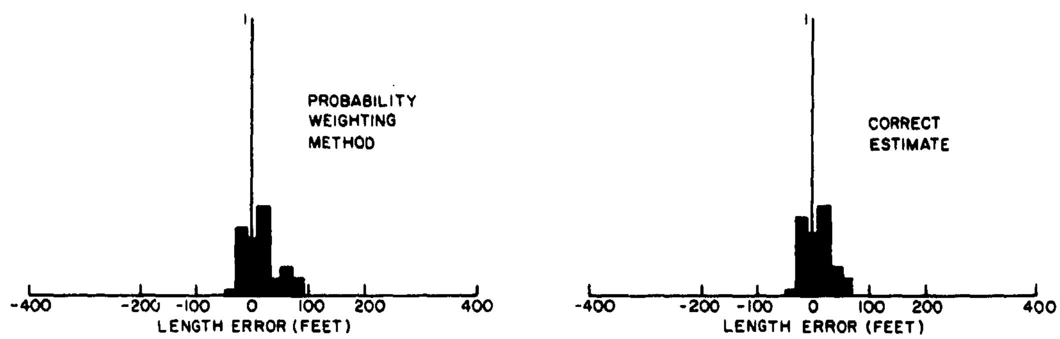


Fig. 12—Relative frequency of the length error for the trawler

Two variations of this basic method were considered. The variation that uses the ship's length and width for the projections is better than the variation which uses only the ship's length. The following conclusions can be made about the variation which uses the ship's width:

1. The ship dimension information is independent of ship size. The standard deviation of the estimate of the ship's length is about 30 ft and over 90 percent of the estimates are within 50 ft of the true length.

2. The heading accuracy is inversely proportional to the size of the ship. Very good heading estimates are obtained for the tanker, fair estimates are obtained for the destroyer, and poor estimates are obtained for the trawler. While the "correct estimate" is fairly good for all ships, it becomes progressively harder to choose the correct solution as the ship becomes smaller. The basic problem is that for the smaller ships, the radar cross section is lower, and the signal-to-noise ratio is too small to make accurate estimates of position.

3. Speed information is poor; no useful speed estimates have been obtained. That is, the standard deviation of the speed error is greater than the standard deviation of the distribution which has been assumed for the speed of the ships.

While this two-beam system has yet to be optimized, the authors believe that very little improvement other than removing the bias can be made on the numbers given in Table 8 and summarized above. Consequently, to improve the speed and heading estimates, a major modification of the system must be made. One possible modification could be the addition of a third beam. With this system, there are only two (instead of four) possible solutions; hence, it would be easier to select the correct solution. Also, by using the heading, the three measured ranges, and a method indicated in Ref. 3, it *may* be possible to obtain useful speed estimates from this system.

REFERENCES

1. G.V. Trunk, "Estimation of Ship Parameters from a Noncoherent Two-Look Side-Looking Radar," NRL Report 7117 (Secret Report, Unclassified Title), Aug. 1970.
2. G.V. Trunk, "Ocean Surveillance: Statistical Considerations," NRL Report 6804 (Secret Report, Unclassified Title), Nov. 1968.
3. J.P. Barry, "The Determination of Target Course and Speed from Radar Data," NRL Report 6807 (Secret Report, Unclassified Title), Nov. 1968.

APPENDIX A

METHOD OF ESTIMATING PROJECTIONS

The obvious estimate of the projection of the target is

$$P = N\Delta, \tag{A1}$$

where Δ is the range resolution of the radar and N is the number of range cells in which the target is detected. However, this is not a good estimate because a point target will pass through several range cells as the radar sweeps by the target. This is due to the fact that the range cells are curvilinear. Specifically, the number of range cells that a stationary point target moves through is

$$M = R_1 [1 - \cos(\theta/2)] / \Delta, \tag{A2}$$

where θ is the radar beamwidth. Consequently, P could be overestimated by as much as $M\Delta$ if Eq. (A1) was used to estimate P. To avoid this error, either of two methods can be used. Since these two estimation methods are rather complicated, the philosophy behind the estimation methods will be presented initially.

As previously mentioned, the trouble with Eq. (A1) is that a point target passes through several range cells as the radar sweeps past the target. Consequently, the estimation method should be based on only a small number of pulses within the beamwidth of the radar so the target does not pass through range cells. Therefore, as soon as a target is detected, the next 50 returned pulses are summed for each of the 25 range cells immediately preceding and following the cell in which the target was detected, the sum for the ℓth cell being denoted by Q_ℓ. As seen in Fig. A1, the middle N cells contain possible returned signal and other cells contain noise. These cells are then used to find the average signal level $\bar{S}$, the average noise level $\bar{Q}$, and the standard deviation of the noise σ_Q. A new threshold $T' = \bar{Q} + 2\sigma_Q$ is defined and is used to detect the target in the N possible cells that contain the target. Finally, end corrections are applied to take care of the situation in Fig. A2 where the target lies only partially within a range cell. The mathematics for the estimation methods is given in the following paragraphs.

Method I

Before explaining the estimation method, it is necessary to describe briefly the target detection system. The detection system in the radar is a feedback integrator. That is, the output for the ith pulse for the jth range cell out of the feedback integrator is

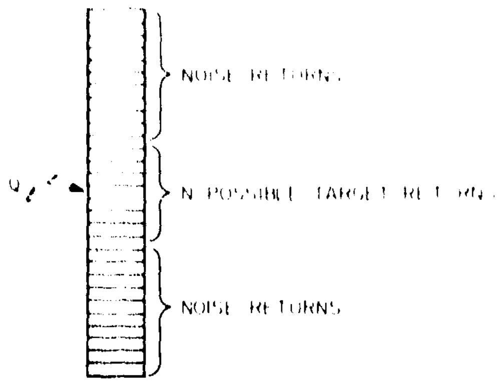


Fig. A1—Summed pulses around target

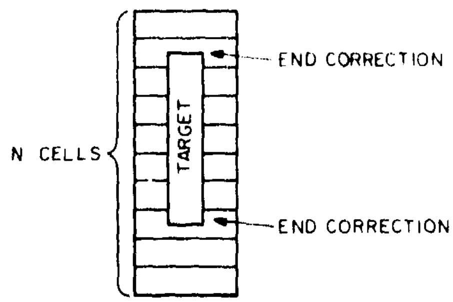


Fig. A2—Target lying partially within a range cell

$$Z_{i,j} = KZ_{i-1,j} + P_{i,j}, \tag{A3}$$

where K is the feedback value and $P_{i,j}$ is the ith returned pulse in the jth range cell. In this system, a target is detected when $Z_{i,j}$ is greater than a threshold T. Let I be the smallest i such that $Z_{i,j} > T$ and let J be the range cell in which the target is detected on the Ith pulse. Then, the following sums Q_ℓ are calculated and saved

$$Q_\ell = \sum_{i=1}^{I_M} P_{I+i,J+\ell-26}, \ell = 1{,}51, \tag{A4}$$

where I_M is the number of pulses used in the sum.* Now, define J_1 and J_2 as the smallest and largest j such that $Z_{i,j} > T$ for any i and let $J_1' = 26 + J_1 - J$ and $J_2' = 26 + J_2 - J$. Obviously, the target is detected in $N = J_2 - J_1 + 1$ range cells. An estimate of the average noise level in the vicinity of the target is

*The value I_M must be a compromise. A large value is needed to obtain good estimates of noise and signal strengths; however, a small value is needed so that the target does not pass through different range cells. For all cases investigated in this report, $I_M = 50$. In Ref. 1, $I_M = 10$.

$$\overline{Q} = \frac{1}{(50 + J_1' - J_2')} \left(\sum_{\ell=1}^{J_1'-1} Q_\ell + \sum_{\ell=J_2'+1}^{51} Q_\ell \right), \tag{A5}$$

and an estimate of the noise variance σ_Q^2 is

$$\sigma_Q^2 = \frac{1}{(50 + J_1' - J_2')} \left[\sum_{\ell=1}^{J_1'-1} (Q_\ell - \overline{Q})^2 + \sum_{\ell=J_2'+1}^{51} (Q_\ell - \overline{Q})^2 \right]. \tag{A6}$$

A new threshold T' is given by

$$T' = \overline{Q} + 2\sigma_Q, \tag{A7}$$

and two integers L_1 and L_2 are defined as the smallest and largest ℓ such that $Q_\ell > T'$. Then, an estimate of the average signal strength is

$$\overline{S} = \frac{1}{(L_2 - L_1 + 1)} \sum_{\ell=L_1}^{L_2} S_\ell, \tag{A8}$$

where $S_\ell = Q_\ell - \overline{Q}$. Finally, the estimated projection is given by

$$\begin{aligned} P = \Delta[(L_2 - L_1 + 1) &+ (V_1 - 1)(\overline{S} - S_{L_1})/\overline{S} + V_1 S_{L_1 - 1}/\overline{S} \\ &+ (V_2 - 1)(\overline{S} - S_{L_2})/\overline{S} + V_2 S_{L_2+1}/\overline{S}], \end{aligned} \tag{A9}$$

where

$$V_1 = \begin{cases} 1 & \text{if } S_{L_1} \geqslant \overline{S} \\ 0 & \text{else} \end{cases}$$

and

$$V_2 = \begin{cases} 1 & \text{if } S_{L_2} \geqslant \overline{S} \\ 0 & \text{else} \end{cases}$$

The final four terms in Eq. (A9) are correction terms which take into account the fact that the target may only partially be in the initial and final range cells (Fig. A2).

Method II

This method is very similar to Method I. The only difference is that I and J are chosen such that $P_{I,J} \geq P_{i,j}$ for all i and j such that $Z_{i,j} > T$. Except for this difference, Method II is the same as Method I. Originally, when I_M was assigned a value of 10, Method II had a higher signal-to-noise ratio than Method I; consequently, slightly better estimates were obtained with Method II. However, when $I_M = 50$, the signal-to-noise ratios are about the same for the two methods, thus, Method I would be preferred since it can be more easily implemented.

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FURTHER RESULTS ON ESTIMATION OF SHIP PARAMETERS [redacted]

This is a final report on one approach; other approaches to the solution of the problem are yet to be examined

Jon David Wilson [redacted] Gerard V. Trunk

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(S) A method has been devised which is capable of estimating a ship's heading and length with a noncoherent side-looking radar possessing two beams—one squinted forward and the other aft. This method uses the ship's projections on the two squinted beams for the estimation. Unfortunately, besides the correct estimate, three spurious pairs of estimates are given. This ambiguity is removed by estimating the target's position in each squinted beam and then using the target's estimated velocity, which is derived from the two positions, to select one of the four estimates. Then, by using a Monte Carlo method, results are obtained on the accuracy of the estimation method. For a typical destroyer with a 450-ft length, the standard deviations of the errors in length and heading are approximately 30.0 ft and 14.0° [redacted].

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